

$$f'(x) = a \cos x - b \sin x \quad f'\left(\frac{4}{3}\pi\right) = 0 \Rightarrow$$

$$a \cos \frac{4\pi}{3} - b \sin \frac{4\pi}{3} = 0 \quad -\frac{1}{2}a + \frac{\sqrt{3}}{2}b = 0$$

$$a - \sqrt{3}b = 0$$

$$f\left(\frac{2}{3}\pi\right) = 1 \quad a \sin\left(\frac{2}{3}\pi\right) + b \cos\left(\frac{2}{3}\pi\right) = 1$$

$$\frac{\sqrt{3}}{2}a - \frac{1}{2}b = 1 \quad \sqrt{3}a - b = 1$$

Resolvamos o sistema

$$\begin{cases} a - \sqrt{3}b = 0 \\ \sqrt{3}a - b = 1 \end{cases} \quad \begin{cases} \sqrt{3}a - 3b = 0 \\ \sqrt{3}a - b = 1 \end{cases}$$

$$\left. \begin{aligned} b &= \frac{1}{2} \\ a &= \frac{\sqrt{3}}{2}b \end{aligned} \right\} \rightarrow$$

$$f(x) = \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x$$

O período de $f(x)$ é 2π